$$= \frac{1}{a+j\omega}\left(e^{-\infty} - e^{0}\right)$$

$$= \left(\frac{1}{j\omega - a}\right)$$

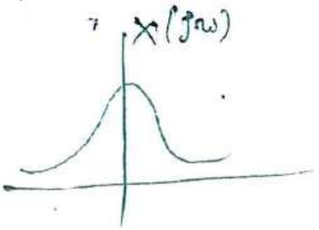
$e^{at} \rightarrow \frac{1}{s-a}$

$e^{-at} \rightarrow \frac{1}{s+a}$

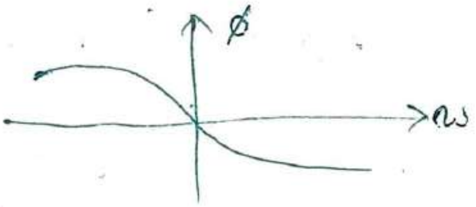
Q. $x(t) = e^{-at}\, u(t)$

$$X(j\omega) = \frac{1}{j\omega + a}$$

$$|X(j\omega)| = \frac{1}{\sqrt{\omega^2 + a^2}} \rightarrow$$



$$\phi = -\tan^{-1}(\omega/a)$$



Q. What is F. transform of

$x(t) = e^{-a|t|}$?

$$X(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= -\int_{0}^{\infty} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left| -\frac{1}{a-j\omega} e^{(a-j\omega)t} \right]_{0}^{\infty} + \left| -\frac{1}{(a+j\omega)} e^{-(a+j\omega)t}\, dt \right|_{0}^{\infty}$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

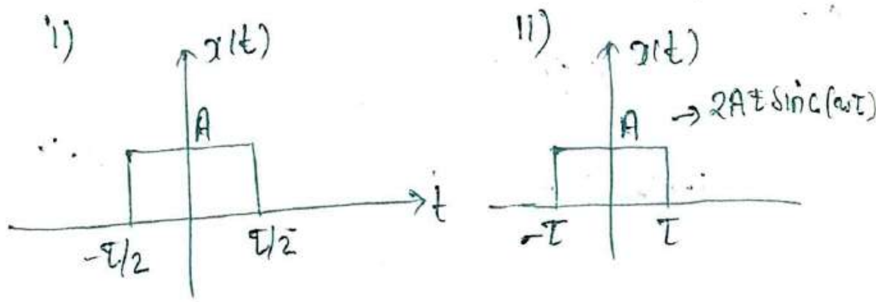
$$= \frac{2a}{a^2 + \omega^2}$$

$$e^{-|t|} \longrightarrow \frac{2}{1+\omega^2}$$

$$e^{-2|t|} \longrightarrow \frac{4}{4+\omega^2}$$

Q. what is F. transform of given fig.

IES-02 i) ii)



(i)

$$X(j\omega) = \int_{-\tau/2}^{\tau/2} A\, e^{-j\omega_0 t}\, dt$$

$$= A \cdot \left| \frac{-1}{j\omega_0} e^{-j\omega_0 t} \right|_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{j\omega} \times 2j \sin(\omega\tau/2)$$

$$= \frac{A\tau}{\omega\tau/2} \sin(\omega\tau/2) = A\tau \,\mathrm{sinc}(\omega\tau/2)$$

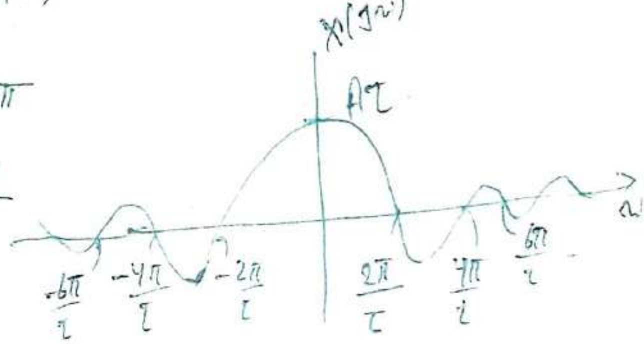
$$X(j\omega) = A\tau \,\mathrm{sinc}(\omega\tau/2)$$

Note:-

1) $A\tau .\mathrm{Sinc}(\omega\tau/2)$

$\omega\tau/2 = \pi$

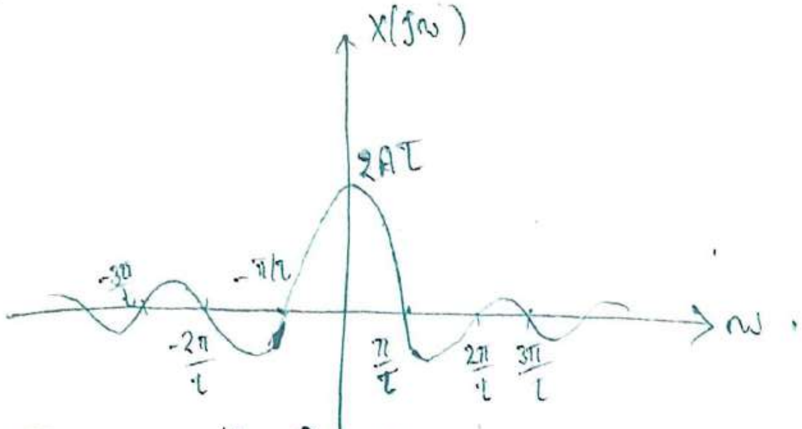
$\omega = \frac{2\pi}{\tau}$



ii) $2A\tau .\mathrm{sinc}(\omega\tau)$

$\omega\tau = \pi$

$\omega = \pi/\tau$



If Time period of pulse increase by 2-time then peak of Sink f^n will be double while freq. will become half.

15/10/07

Fourier transform:-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega.$$

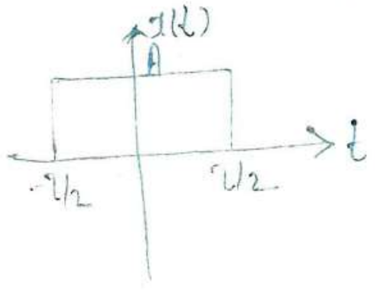
Q. The fourier transform of a voltage signal $x(t)$ is $X(j\omega)$, what is the unit of $|X(j\omega)|$

GATE 90

A) Volt
B) Volt-sec. (✓)
c) Volt/sec.
d) Volt^2

$$X(j\omega) = \int_{-\infty}^{\infty} \underbrace{x(t)}_{\text{Volt}}\ \underbrace{e^{-j\omega t}}_{\text{No unit}}\ \underbrace{dt}_{\text{Sec.}}$$

Q. What is F.T. of given figure?



$$X(j\omega) = \int_{-\tau/2}^{\tau/2} A \cdot e^{-j\omega t}\, dt$$

$$= A\tau\ \mathrm{Sinc}(\omega\tau/2)$$

Q. What is Fourier transform of Impulse f'n

$x(t) = \delta(t)$

Ans.

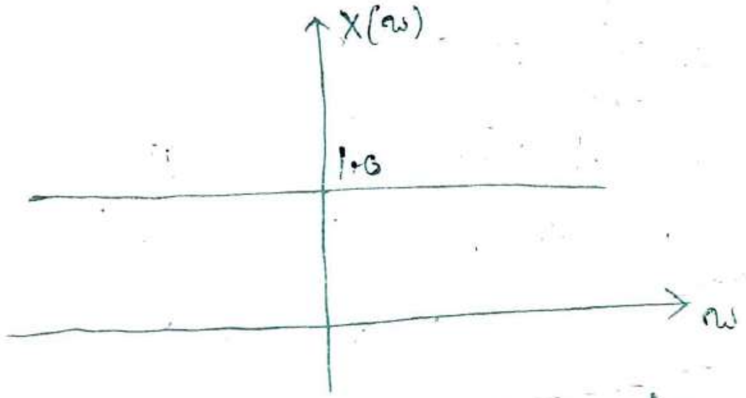
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$ [at $t=0$, $\delta(t)=1$ only defined other value of t, $\delta(t)=0$]

FT → Analysis of Signal in frequency domain.

So, Band width of Impulse f'n is infinity.



for any value of ω, unit impulse function cover the Entire value as shown above i.e B.W of unit impulse f'n ∞.

Q. What is Inverse F.T of

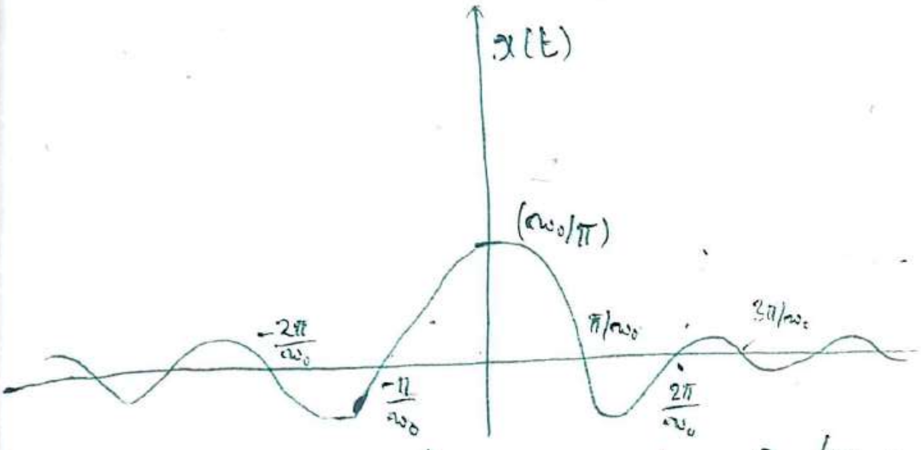
$$X(j\omega) = \begin{cases} 1 & |\omega| < \omega_0 \Rightarrow -\omega_0 \le \omega \le \omega_0 \\ 0, & \text{else where.} \end{cases}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi j t}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

$$= \frac{2j\sin\omega_0 t}{2\pi j t} = \frac{\sin\omega_0 t}{(\pi t)} = \frac{\omega_0}{\pi} \times \left(\frac{\sin\omega_0 t}{\omega_0 t}\right)$$

$$x(t) = \frac{\omega_0}{\pi}\,\mathrm{Sinc}(\omega_0 t)$$



This function has max. value of $\left(\frac{\omega_0}{\pi}\right)$.

Fourier transform of Periodic Signals:-

if $x(t) = e^{j\omega_0 t}$ is periodic signal with period $T_0 = (2\pi/\omega_0)$

S-I - first calculate Fourier series coeff. then calculate fourier transform by formula

$$X(j\omega) = \sum_{K=-\infty}^{\infty} 2\pi C_K\, \delta(\omega - K\omega_0)$$

Q. what is F.T. of f'n $x(t) = 1$. ...

Ans.

$$x(t) = e^{j0\cdot t} = e^{j\omega_0 t}$$

$$\omega_0 = 0 \Rightarrow T = \frac{2\pi}{0} = \text{undefined}$$

$x(t) = 1$ is constant f'n with fundamental period frequency $(\omega_0) = 0$.

$$X(j\omega) = \sum_{K=-\infty}^{\infty} 2\pi\,\delta(\omega - K\omega_0)\,.\,C_K$$

out $\omega_0 = 0$, $\Rightarrow$ $2\pi C_0\,\delta(\omega_0) = 2\pi\,\delta(\omega)$

$1 \rightarrow 2\pi\delta(\omega)$

$A \rightarrow 2\pi A\delta(\omega)$

Q. What is F transform of $x(t) = \cos\omega_0 t$?

Ans.

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - k\omega_0) \cdot C_k$$

$$\cos\omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \qquad \left[e^{jk\omega_0 t} \right]_{k=1}$$

$$(e^{-jk\omega_0 t}) =$$

$$C_1 = \left(\frac{1}{2}\right), \quad C_{-1} = \left(\frac{1}{2}\right)$$

$$X(j\omega) = 2\pi\delta(\omega - \omega_0)C_1 + 2\pi\delta(\omega + \omega_0)C_{-1}$$

$$\boxed{X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)}$$

Q. What is F. transform of $x(t) = \sin\omega_0 t$

$$\sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$C_1 = \left(\frac{1}{2j}\right) \quad . \quad C_{-1} = \left(-\frac{1}{2j}\right).$$

$$X(j\omega) = 2\pi\delta(\omega - \omega_0)C_1 + 2\pi\delta(\omega + \omega_0)C_{-1}$$

$$= \frac{2\pi\delta(\omega - \omega_0) \times 1}{2j} + \frac{2\pi\delta(\omega + \omega_0) \times (-1)}{2j}$$

$$= \frac{\pi}{j}\left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$$

$$= \pi j\left[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)\right].$$

So, fourier transform of periodic impulse train in time domain is also impulse train in frequency domain with time period $\frac{2\pi}{T}$ i.e, spacing b/w impulse ~~is~~ in freq. domain get smaller.

Important property of F.T :-

1. $x(t) \xrightarrow{F.T.} X(j\omega)$

   $x(t-t_0) \longrightarrow e^{-j\omega t_0} X(j\omega)$

2. $x(t) \xrightarrow{F.T.} X(j\omega)$

   $e^{j\omega_0 t} x(t) \xrightarrow{F.T.} X(j(\omega-\omega_0))$

3. $x(t) \xrightarrow{F.T.} X(j\omega)$

   $\frac{dx(t)}{dt} \xrightarrow{F.T.} j\omega X(j\omega)$

   $\frac{d^n x(t)}{dt^n} \xrightarrow{F.T.} (j\omega)^n X(j\omega)$

4. $x(t) \xrightarrow{F.T.} X(j\omega)$

   $$\int_{-\infty}^{t} x(\tau)\,d\tau \longrightarrow \frac{1}{j\omega} X(j\omega) + \pi \underbrace{X(0)}_{\text{F.T. of } x(t)} \delta(\omega),$$

Q. calculate F.T of $x(t) = u(t)$?

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau \longrightarrow \cancel{\frac{1}{j\omega}\delta(j\omega)}$$

$$= \frac{1}{j\omega}\delta(j\omega) + \pi\delta(0)\,\delta(\omega)$$

$$= \frac{1}{j\omega} + \pi\delta(\omega)$$

$x(t) = u(t)$

$\dot{x}(t) = j\omega\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]$ $\quad\left[\because \omega\delta(\omega) = 0,\ t\delta(t) = 0\right]$

$= 1 + \pi(\delta(\omega))\,j\omega$

$= 1$

Q. $x(t) = sgn(t)$, find F.T ?

Ans. $sgn(t) = \begin{cases} 1, & t>0 \\ -1, & t<0 \end{cases}$

$sgn(t) = 2u(t) - 1$

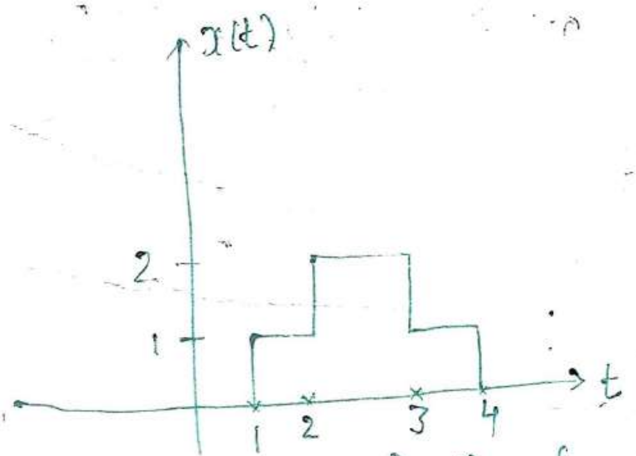
$F[sgn(t)] = 2F[u(t)] - F[1]$

$= 2\left[\frac{1}{j\omega} + \pi\delta(\omega)\right] - 2\pi\delta(\omega)$

$= \frac{2}{j\omega} + 2\pi\delta(\omega) - 2\pi\delta(\omega)$

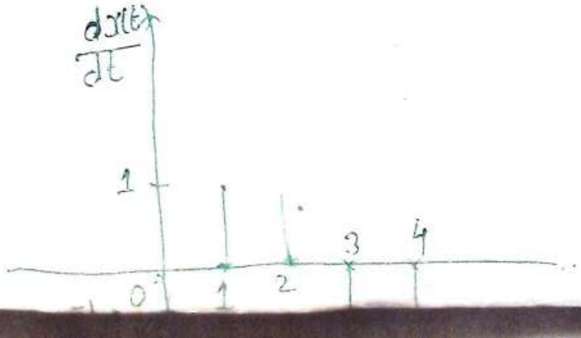
$$\boxed{F[sgn(t)] = 2/j\omega}$$

Q. IES-05



Calculate F.T of this figure ?

Ans.



$$\frac{dx}{dt} = \delta(t-1) + \delta(t-2) - \delta(t-3) - \delta(t-4)$$

$$\delta(t) \xrightarrow{F.T} 1$$

$$\delta(t-t_0) \xrightarrow{F.T} 1 \cdot e^{-j\omega t_0}$$

$$\delta(t-1) \xrightarrow{F.T} e^{-j\omega}$$

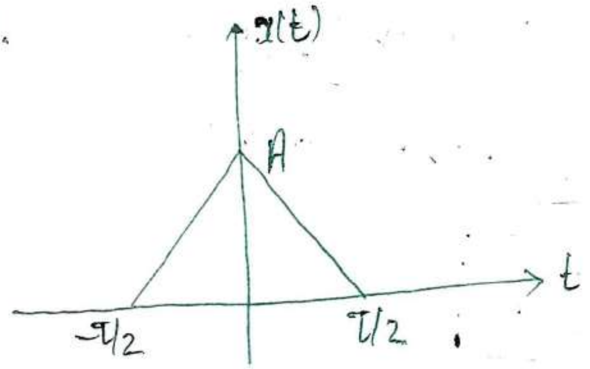
$$\frac{dx(t)}{dt} = \cancel{0}$$

$$j\omega\, X(j\omega) = e^{-j\omega} + e^{-2j\omega} - e^{-3j\omega} - e^{-4j\omega}$$

$$= e^{-j\omega}(1 + e^{-2j\omega}) - e^{-3j\omega}(1 + e^{-2j\omega})$$

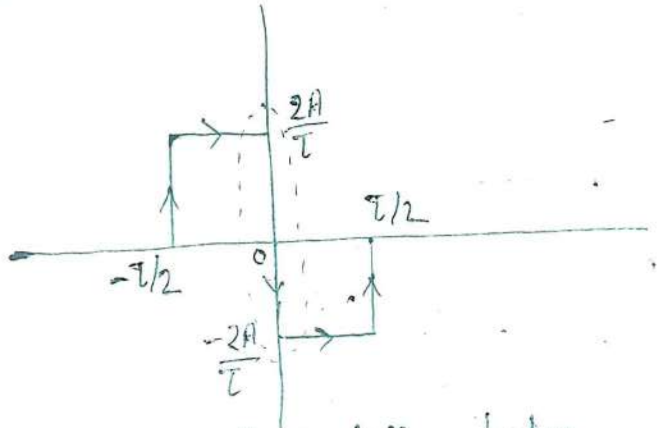
$$= (e^{-j\omega} - e^{-3j\omega})(1 + e^{-2j\omega})$$

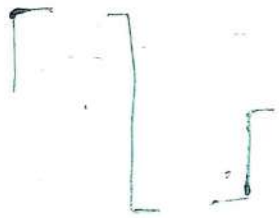
Q.



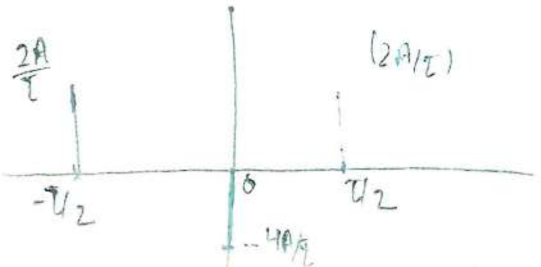
Ans. ON differentiation

$$x(t) = \begin{cases} \frac{2A}{\tau} t & -\frac{\tau}{2} < t < 0 \\ \frac{2A}{\tau}(T - t), & 0 < t < \frac{\tau}{2} \end{cases}$$





ON further differentiation



$$\frac{d^2x(t)}{dt^2} = \frac{2A}{\tau}\delta(t+\tau/2) - \frac{4A}{\tau}\delta(t) + \frac{2A}{\tau}\delta(t$$

$$(j\omega)^2 X(j\omega) = \frac{2A}{\tau}e^{j\omega\tau/2} - \frac{4A}{\tau} + \frac{2A}{\tau}e^{-j\omega\tau/2}$$

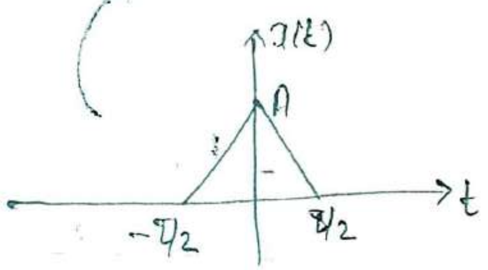
$$= \frac{4A}{\tau}\cos(\omega\tau/2) - \frac{4A}{\tau}$$

$$\omega^2 X(j\omega) = \frac{4A}{\tau}(1-\cos\omega\tau/2)$$

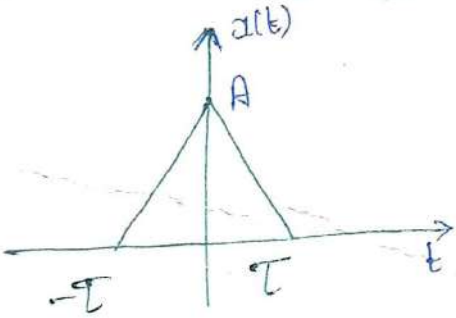
$$= \frac{4A}{\omega^2\tau}\, 2\sin^2(\omega\tau/4)$$

$$= \cancel{\frac{8A}{\omega^2\tau}\sin^2(\omega\tau/4)}\left(\frac{A\tau}{2}\right)\times\frac{\sin^2(\omega\tau/4)}{(\omega\tau/4)^2}$$

$$\boxed{X(j\omega) = \frac{A\tau}{2}\,\text{sinc}^2(\omega\tau/4)}$$

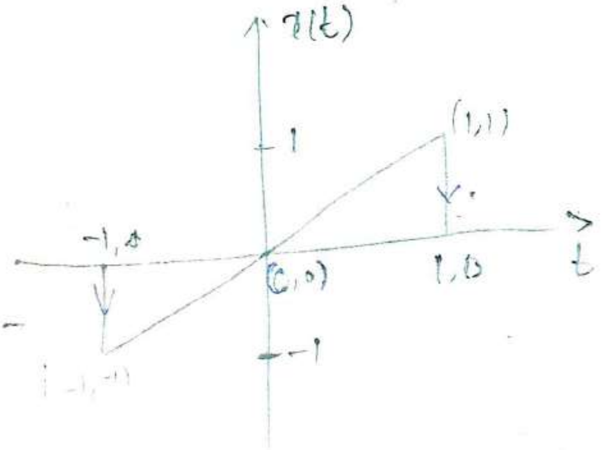


Q. What is F.T of fig.



Ans. $X(j\omega) = A\tau\,\text{sinc}^2(\omega\tau/2)$

Q.
*



Q. What is the F.T of

$$x(t) = \frac{2}{1+t^2}$$

Soln

A) $2\pi e^{-2|\omega|}$ B) $2\pi e^{-|\omega|}$

C) $2\pi e^{-\omega/2}$ d) $2\pi(\tan^{-1}\omega)$

Soln

$$e^{-a|t|} \xrightarrow{F.T} \frac{2a}{a^2+\omega^2}$$

$$e^{-|t|} \xrightarrow{F.T.} \frac{2}{1+\omega^2}$$

$$\frac{2}{1+t^2} \longrightarrow 2\pi e^{-|\omega|}$$

Q. what is F.T of

$$x(t) = \frac{1}{a^2+t^2}$$

Ans.

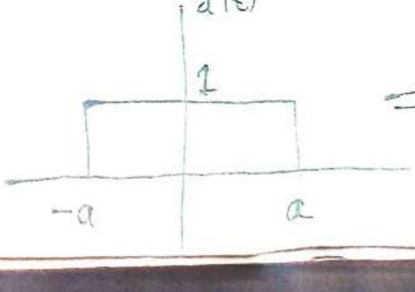
$$\Rightarrow e^{-a|t|} \longrightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{1}{a^2+t^2} \longrightarrow \frac{\pi}{a} e^{-a|\omega|}$$

Q. what is F.T of $x(t) = \left(\frac{\sin at}{\pi t}\right)$

Ans.

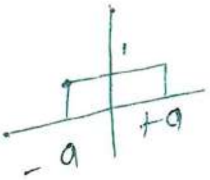


$$\Rightarrow X(j\omega) = \int_{-a}^{a} 1 \cdot e^{-j\omega t}\, dt$$

$$= -\frac{1}{j\omega}\left(e^{-j\omega a} - e^{j\omega a}\right)$$

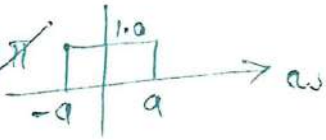
$$= \left(\frac{2 \sin \omega a}{\omega} \right)$$

$$X(j\omega) \quad - \quad \frac{2\pi \sin \omega a}{\omega \pi}$$

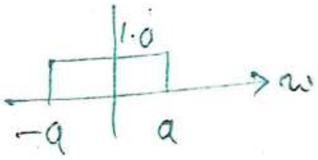


$$\xrightarrow{F.T.} \frac{2\pi \sin \omega a}{\omega \pi}$$

$$\frac{\cancel{2\pi} \sin at}{\pi t} \longrightarrow \cancel{2\pi}$$



$$\frac{\sin at}{\pi t} \xrightarrow{F.T.}$$



Q. $x[n] = \begin{cases} 1 & 0 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases}$

$h[n] = \begin{cases} \alpha^n, & 0 \leq n \leq 6 \\ 0, & \text{otherwise} \end{cases}$

$0 \leq n \leq 4$

$0 \leq n \leq 6$

$0, 6, 4, 10, \cancel{4}, \cancel{6}$

$0 \leq n \leq 4$

$4 \leq n \leq 6$

$6 \leq n \leq 10$

Q. $x(n) = \begin{cases} 1 & 3 \le n \le 8 \\ 0, & \text{elsewhere} \end{cases}$

$h[n] = \begin{cases} 1, & 4 \le n \le 15 \\ 0, & \text{elsewhere} \end{cases}$

7, 18, 12, 23, 11, 19

first add, cross multiply all range value in convolution and the range come will be non-zero while other range value be zero.

$\left.\begin{array}{l} 7 \le n \le 11 \\ 12 \le n \le 18 \\ 19 \le n \le 23 \end{array}\right\}$ Non zero

16/10/07

Q. What is F. transform of

1. $x(t) = u(t)\cos\omega_0 t$

Ans.

$$F.T\ X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\int_0^{\infty} \cos\omega_0 t \cdot e^{-j\omega t} dt = \int_0^{\infty} \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) \times e^{-j\omega t} d$$

$$= \frac{1}{2}\int_0^{\infty} \left(e^{jt(\omega_0-\omega)} + e^{-jt(\omega_0+\omega)}\right) dt$$

$$= \frac{1}{2}\int_0^{\infty} e^{-jt(\omega-\omega_0)} dt + \frac{1}{2}\int_0^{\infty} e^{-jt(\omega_0+\omega)} dt$$

$$= -\frac{1}{2j(\omega-\omega_0)}\left(e^{-\infty} - e^{0}\right) + \frac{1}{2}\left(e^{-\infty} - e^{0}\right) \times \frac{1}{-j(\omega+\omega_0)}$$

$$= \frac{-1}{2j(\omega_0-\omega)} + \frac{1}{2j(\omega+\omega_0)}.$$

$$u(t)\cos\omega_0 t \xrightarrow{F.T.} \frac{1}{2j(\omega+\omega_0)} + \frac{1}{2j(\omega-\omega_0)} =$$

$$u(t)\sin\omega_0 t \xrightarrow{F.T.} \frac{1}{2(\omega-\omega_0)} - \frac{1}{2(\omega+\omega_0)} =$$

Q. calculate F. transform of $u(t)\sin\omega_0 t$?

Ans.

$$\int_0^\infty \left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) e^{-j\omega t}\,dt$$

$$= \frac{1}{2j}\int_0^\infty \left(e^{jt(\omega_0-\omega)} - e^{-jt(\omega+\omega_0)}\right) dt$$

$$= \frac{1}{2j}\int_0^\infty e^{-jt(\omega-\omega_0)}\,dt - \frac{1}{2j}\int_0^\infty e^{-jt(\omega+\omega_0)}\,dt$$

$$= \frac{-1}{2j(\omega-\omega_0)j}\left(e^{-\infty} - e^0\right) - \frac{1}{2j}\frac{\left(e^{-\infty} - e^0\right)}{-j(\omega+\omega_0)}.$$

$$= \frac{1}{2(\omega-\omega_0)} - \frac{1}{2(\omega+\omega_0)}.$$

Q. calculate F. transform of $e^{-at}\cos\omega_0 t\, u(t)$?

Ans.

$$= \int_0^\infty e^{-at}\left(\frac{e^{j\omega_0 t} \pm e^{-j\omega_0 t}}{2}\right) dt$$

$$= \int_0^\infty e$$

$$e^{-at}u(t) \longrightarrow \frac{1}{j\omega+a}$$

$e^{-at}\cos\omega_0 t\, u(t)$

$$e^{-at}\cdot\frac{(e^{j\omega_0 t}+e^{-j\omega_0 t})}{2}$$

$$x(t) = \frac{e^{-(a-j\omega_0)t}+e^{-(a+j\omega_0)t}}{2}$$

$$=\frac{1}{2}\left[\frac{1}{j\omega+a-j\omega_0}+\frac{1}{j\omega+a+j\omega_0}\right]$$

Q. $\cos\omega_0 t\, u(t) \longrightarrow \frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}$

$$\frac{1}{2}\left[\frac{1}{j\omega-j\omega_0}+\frac{1}{j\omega+j\omega_0}\right]$$

$$=\frac{1}{2j}\left[\frac{1}{\omega-\omega_0}+\frac{1}{\omega+\omega_0}\right]$$

$$\sin\omega_0 t\, u(t) \longrightarrow \left(\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}\right)u(t)$$

$$\frac{1}{2j}\left[\frac{1}{j\omega-j\omega_0}-\frac{1}{j\omega+j\omega_0}\right]$$

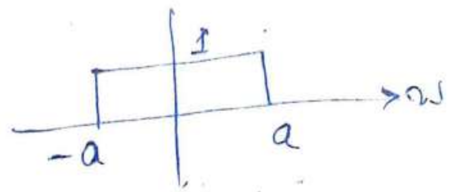
$$\frac{1}{2j^2}\left[+\frac{1}{\omega-\omega_0}+\frac{1}{\omega+\omega_0}\right]$$

$$=\frac{1}{2}\left[-\frac{1}{\omega+\omega_0}+\frac{1}{\omega-\omega_0}\right]$$

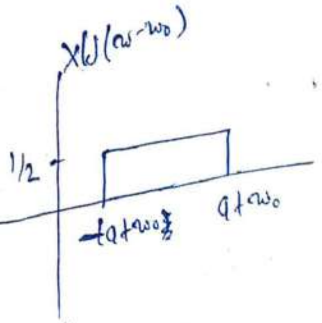
Q. What is FT of

$$x(t) = \frac{\sin at}{\pi t}(\cos \omega_0 t)$$

Ans. $\dfrac{\sin at}{\pi t} \xrightarrow{F.T.}$

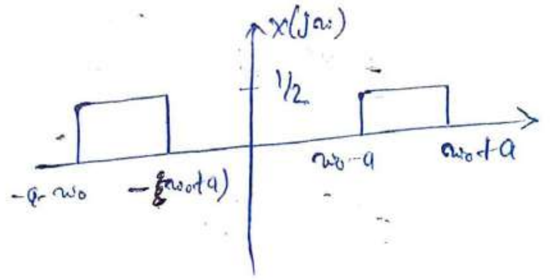


$$= \frac{\sin at}{\pi t}\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right)$$



$$\left[\begin{array}{l} e^{j\omega_0 t} x(t) \rightarrow X(j(\omega-\omega_0)) \\ e^{-j\omega_0 t} x(t) \rightarrow X(j(\omega+\omega_0)) \end{array}\right]$$

$=$



Parseval's Relationship —

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$$

Energy of a signal.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

$$X(0) = \int_{-\infty}^{\infty} x(t) \, dt$$

This Equ. indicate area of Signal is Equal to value of fourier transform at $\omega = 0$

Convolution of $x(t)$ & $y(t)$?

$$x(t) \xrightarrow{F.T.} X(j\omega) \quad \text{then}$$

$$y(t) \xrightarrow{F.T.} Y(j\omega)$$

$$x(t) \otimes y(t) \xrightarrow{F.T.} X(j\omega) \cdot Y(j\omega)$$

Convolution in time domain is Equal to multiplication in frequency domain.

Q. what is convolution of 1. $x(t) = e^{-at} u(t)$

2. $y(t) = e^{-bt} u(t)$

$z(t) = x(t) \otimes y(t)$?

Ans.

$$e^{-at} u(t) \xrightarrow{F.T.} \frac{1}{j\omega + a}$$

$$e^{-bt} u(t) \longrightarrow \frac{1}{j\omega + b}$$

$$Z(j\omega) = X(j\omega) \cdot Y(j\omega)$$

$$= \frac{1}{j\omega + a} \cdot \frac{1}{j\omega + b}$$

$$= \frac{1}{(j\omega + a)(j\omega + b)}$$

take the inverse Laplace transform

$$= \frac{1}{a-b} \left[\frac{1}{b + j\omega} - \frac{1}{a + j\omega} \right]$$

$$z(t) = \frac{1}{a-b} \left[e^{-bt} u(t) - e^{-at} u(t) \right]$$

Q. what is convolution of $x(t) = e^{-at}u(t)$
$y(t) = e^{-at}u(t)$

$z(t) = x(t) \otimes y(t)$?

Ans. $X(j\omega) = \dfrac{1}{a+j\omega}$, $Y(j\omega) = \dfrac{1}{a+j\omega}$ $\Rightarrow Z(j\omega) = \dfrac{1}{(a+j\omega)^2}$

$x(t) \longrightarrow X(j\omega)$

$t\,x(t) \longrightarrow j\left(\dfrac{d}{d\omega}X(j\omega)\right)$

$$te^{-at} \longrightarrow j\cdot\frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right) = j\left[\frac{(a+j\omega)\cdot 0 - 1\times j}{(a+j\omega)^2}\right]$$

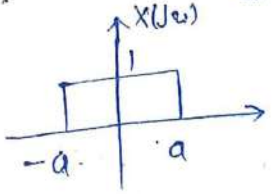
$$= \frac{1}{(a+j\omega)^2}$$

$z(t) = te^{-at}$

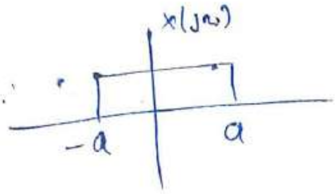
Q. What is convolution of Two Sinc f'n ?

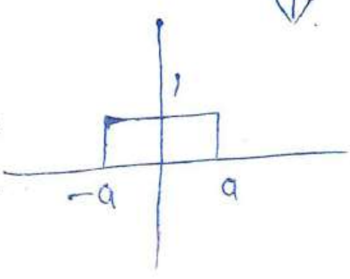
A) sinc (✓)
B) Sinc^2
c) Rectangular
d) Triangular

Sinc $\longrightarrow$



Sinc $\longrightarrow$





Sinc $\xleftarrow{\text{Inverse}}$

Q. What is value of this integration ?

$$y(t) = \int_{-\infty}^{\infty} \frac{\sin x}{x} * \frac{\sin(t-x)}{(t-x)}\, dx = \cancel{\frac{\sin t}{t} \times \frac{\sin}{t}}$$

a) 1 b) ~~Zero~~ Sinc (✓)

c) 0 d) Rectangular

$$x(t) \otimes h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \frac{\sin\tau}{\tau}\, \frac{\sin(x-\tau)}{(x-\tau)}\, d\tau.$$

Q. If $x(t)$ is a signal with fourier transform given by

$$X(j\omega) = \begin{cases} 1, & |\omega| < 1 \\ 0, & \text{Elsewhere} \end{cases}$$

if $y(t) = \frac{d^2 x(t)}{dt^2}$; then calculate

$$\int_{-\infty}^{\infty} |y(t)|^2\, dt$$

Ans. By using parsevall's theorem

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(j\omega)|^2\, d\omega$$

$$Y(j\omega) = (j\omega)^2 X(j\omega)$$

$$= \begin{cases} -\omega^2 , & |\omega| < 1 \\ 0 , & \text{Elsewhere} \end{cases}$$

$$|Y(j\omega)|^2 = (\omega^2 X(j\omega))^2$$

$$= \frac{1}{2\pi} \int_{-1}^{1} \omega^4 \, d\omega$$

$$= \frac{1}{2\pi} \times \frac{1}{5} \times \left[\omega^5\right]_{-1}^{1}$$

$$= \frac{1}{10\pi} \times 2 = \frac{1}{5\pi}$$

Q. $x(t) \xrightarrow{F.T.} X(j\omega)$

$x^*(t) \xrightarrow{F.T.} X^*(j\omega)$

$$X^*(j\omega) = \int_{-\infty}^{\infty} x^*(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X^*(j\omega) = \int_{-\infty}^{\infty} x^*(t) e^{j\omega t} dt$$

1. If $x(t)$ is pure real

$$x^*(t) = x(t)$$

$$\therefore X^*(j\omega) = \int_{-\infty}^{\infty} x(t) e^{+j\omega t} dt$$

$$= X(-j\omega)$$

2. If $x(t)$ is pure imaginary

$$x^*(t) = -x(t)$$

$$X^*(j\omega) = \int_{-\infty}^{\infty} -x(t) e^{j\omega t} dt$$

$$= -X(-j\omega)$$

$x(t) \xrightarrow{F.T.} X(j\omega)$

$x^*(t) \xrightarrow{F.T.} X^*(j\omega)$ — $X(-j\omega)$ if $x(t)$ is real
— $-X(-j\omega)$ if $x(t)$ is imaginary.

1. If $x(t)$ is real & Even

$$X^*(j\omega) \longrightarrow X(-j\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t}\, dt$$

Put $t = -y$

$dt = -dy$

$$= -\int_{+\infty}^{-\infty} x(-y)\, e^{-j\omega y}\, dy$$

$$= -\int_{+\infty}^{-\infty} x(y)\, e^{-j\omega y}\, dy$$

$$= \int_{-\infty}^{\infty} x(y)\, e^{-j\omega y}\, dy$$

$$= X(j\omega)$$

If $x(t)$ is real & Even then F. transform $X(j\omega)$ is real & Even.

2. If $x(t)$ is real & odd.

$x(t) \longrightarrow X(j\omega)$

$$x^*(t) \longrightarrow X^*(j\omega) = X(-j\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t}\, dt$$

$$= -\int_{+\infty}^{-\infty} x(-y)\, e^{-j\omega y}\, dy$$

$$= -\int_{-\infty}^{\infty} x(y)\, e^{-j\omega y}\, dy$$

$$= -X(j\omega)$$

If $x(t)$ is real & odd then F. transform Imaginary & odd.

3. $x(t)$ is imaginary & Even

$$x(t) \longrightarrow X(j\omega)$$

$$x^*(t) \longrightarrow X^*(j\omega) = -X(-j\omega)$$

$$= -\int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, dt$$

put $t = -\tau$

$dt = -d\tau$

$$= -\int_{+\infty}^{-\infty} x(-\tau)\, e^{-j\omega\tau}(-d\tau)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= -X(j\omega).$$

$$X^*(j\omega) = -X(-j\omega) \rightarrow -X(j\omega)$$

$$X(-j\omega) - X(j\omega)$$

F. transform is imaginary & ~~odd~~. Even

4. $x(t)$ is imaginary & odd.

then its F. transform is real & odd.

$x(t)$		$X(j\omega)$	
i) Real	Even	Real	Even
ii) Real	odd	Imaginary	odd
iii) Imaginary	Even	Imaginary	Even
iv) "	odd	Real	odd

Note:- If f'n is even then nature of $X(j\omega)$ will be same as $x(t)$. but if f'n is odd then real will change to imaginary & vice-versa.

Q. The F. transform of a conjugate symmetric function is always
GATE 2004

A) Imaginary
B) Real (✓)
c) conjugate & Anti-Symmetric
d) " & Symmetric

$X^*(j\omega) = X(j\omega) \rightarrow$ Conjugate Symmetric

Even
$$\begin{cases} X^*(-j\omega) = X^*(j\omega) \\ \text{on taking conjugate} \\ X(-j\omega) = X(j\omega) \end{cases}$$

Q. If $G(f)$ of a signal $g(t)$, which is real & odd symmetric in time, then

A) $G(f)$ is complex

B) $G(f)$ is imaginary (✓)

c) " is real.

d) $G(f)$ is real & Non, negative

Q. If $X(j\omega) = e^{-2|\omega|}$
what is value of $x(t)$?

Ans.

$$\cancel{\frac{2a}{a^2+\omega^2}} \rightarrow e^{-a|t|} \rightarrow \frac{2a}{a^2+\omega^2}$$

$$\frac{2a}{a^2+t^2} \longrightarrow 2\pi e^{-a|\omega|}$$

$$\frac{a/\pi}{a^2+t^2} \longrightarrow e^{-a|\omega|}$$

put $a=2$, we get

$$\Rightarrow \frac{2}{\pi(4+t^2)}$$

Q. what is $x(t)$ if $X(j\omega) = e^{-2\omega} u(\omega)$?

Ans.

$$e^{-at}u(t) \longrightarrow \frac{1}{a+j\omega}$$

$$\frac{1}{a+jt} \longrightarrow 2\pi e^{+a\omega} u(-\omega)$$

$$\frac{1}{a-jt} \longrightarrow 2\pi e^{-a\omega} u(\omega)$$

$$\frac{1}{2\pi(a-jt)} \longrightarrow e^{-a\omega} u(\omega)$$

$x(t) \longrightarrow X(\omega)$

$X(t) \longrightarrow 2\pi x(-\omega)$

Q. If $x(t) = X(j\omega)$ what is F. transform of x

Sol.ⁿ $x(3(t-2)) = \frac{1}{3} X(j\omega/3) \times e^{-2j\omega}$ $x(at) \longrightarrow \frac{1}{|a|} X(j\omega$

$x(t-t_0) \longrightarrow e^{-j\omega t_0} X$

Q. For a signal $x(t)$ if F.T. is $X(f)$ then what is inverse F.T. of $X(3f+2)$?

A) $\frac{1}{2} x(t/2) e^{j3\pi t}$ B) $\frac{1}{3} x(t/3) e^{-j4\pi t/3}$

c) $3x(3t) e^{-j4\pi t}$ d) $x(3t+2)$

Ans. $X(3(f+2/3)) = \frac{1}{3} x(t/3) e^{-j2\pi t \times (2/3)}$

$= \frac{1}{3} x(t/3) e^{-j4\pi t/3}$

→ tech. (